CBCS SCHEME

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15MAT11

First Semester B.E. Degree Examination, Feb./Mar. 2022

Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the n^{th} derivative of $e^{ax}\cos(bx + c)$ **(05 Marks)**

b. Show that the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ cut orthogonally. **(05 Marks)**

c. Show that the radius of curvature at (a, 0) on the curve $y^2x = a^2(a - x)$ is a/2. **(06 Marks)**

OR

2 a. Find the n^{th} derivative of

$$\frac{x}{(x-1)^2(x+2)}$$ **(05 Marks)**

b. If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ prove that $x^2y_{n+2} + (2n + 1)xy_{n+1} + 2n^2y_n = 0$ **(06 Marks)**

c. Find the angle between the radius vector and the tangent for the curve $r^m = a^m(\cos m\theta + \sin m\theta)$ **(05 Marks)**

Module-2

3 a. Evaluate $\lim_{x \to 0} \frac{\sinh x - \sin x}{x \sin^2 x}$ **(05 Marks)**

b. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 = \frac{-9}{(x+y+z)^2}$ **(06 Marks)**

c. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$ find $J = \frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ **(05 Marks)**

OR

4 a. Obtain Maclaurin's expansion of $\log(1 + e^x)$ **(05 Marks)**

b. Evaluate $\lim_{x \to 0}\left(\frac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$ **(06 Marks)**

c. If $u = F(x - y, y - z, z - x)$ prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(05 Marks)**

Module-3

5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of velocity and acceleration at $t = 1$ in the direction of $\hat{i} - 3\hat{j} + 2\hat{k}$. **(05 Marks)**

b. Find the constant a, so that $\vec{F} = y(ax^2 + z)\hat{i} + x(y^2 - z^2)\hat{j} + 2xy(z - xy)\hat{k}$ is solenoidal. **(05 Marks)**

c. Prove that $\nabla \times (\phi\vec{A}) = \phi(\nabla \times \vec{F}) + (\nabla\phi) \times \vec{F}$ **(06 Marks)**

OR

6 a. Find the directional derivative of $\phi(x.y.z) = x^2yz + 4xz^2$ at the point (1, –2, –1) in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$. **(05 Marks)**

b. Show that the vector field $\vec{F} = (z + \sin y)\hat{i} + (x\cos y - z)\hat{j} + (x - y)\hat{k}$ is irrotational. Also find the scalar function ϕ such that $\vec{F} = \nabla\phi$. **(06 Marks)**

c. Prove that $\text{div}(\text{curl}\,\vec{F}) = 0$ **(05 Marks)**

Module-4

7 a. Obtain the reduction formula of $\int_0^{\pi/2} \sin^n x\,dx$ **(05 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$ **(05 Marks)**

c. Show that the family of ellipses $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$ is self-orthogonal. (a and b are constants and λ is parameter). **(06 Marks)**

OR

8 a. Evaluate $\int_0^{\pi} x\sin^4 x\cos^2 x dx$ **(05 Marks)**

b. Solve $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$. **(06 Marks)**

c. Show that the family of curves $y^2 = 4a(n + a)$ is self orthogonal. **(05 Marks)**

Module-5

9 a. Find the rank of the matrix by reducing it to echelon form. Given

$$A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$$ **(05 Marks)**

b. Solve the following system of equation by Gauss-Seidel method.

$20x + y - 2z = 17$
$3x + 20y - z = -18$
$2x - 3y + 20z = 25$ **(06 Marks)**

c. Use power method to find the largest eigen value and the corresponding vector

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \quad X_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$ **(05 Marks)**

OR

10 a. Solve by Gauss elimination method

$x + 2y + z = 3$
$2x + 3y + 2z = 5$
$3x - 5y + 5z = 2$ **(05 Marks)**

b. Show that the transformation

$y_1 = 2x_1 - 2x_2 - x_3$
$y_2 = -4x_1 + 5x_2 + 3x_3$
$y_3 = x_1 - x_2 - x_3$

is regular and find the inverse transformation. **(05 Marks)**

c. Reduce the Quadratic form

$3x_1^2 + 3x_2^2 + 3x_3^3 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ into canonical form and indicate the nature, rank, index and signature of the Quadratic form. **(06 Marks)**